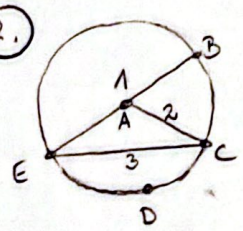
KRUG i KRUŽNICA – rješenja zadataka

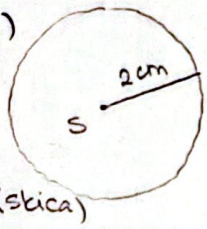
① Definicije matematičkih pojmova mogu se naći u udžbeniku!

②



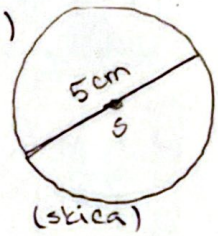
a) A → središte kruga/kružnice
b) $\overline{CE}$ → tetiva
c) $\overline{BE}$ → promjer
d) $\overline{AB}$ → polumjer
e) $\overset{\frown}{CB}$ → kružni luk nad kutom $\sphericalangle CAB$
f) $\overset{\frown}{BE}$ → kružni luk nad kutom $\sphericalangle BAE$ (polukružnica)
g) polukrug
h) kružni isječak
i) kružni odsječak

③ a)



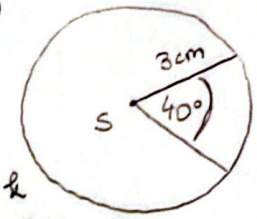
(skica)

b)



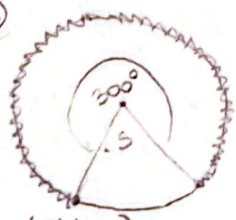
(skica)

④



(skica)

⑤



(skica)

⑦ a) $|\sphericalangle CSA| = ?$

$|\sphericalangle ASB| = 115°$

$|\sphericalangle BSC| = 70°$

$|\sphericalangle CSA| = 360° - (70° + 115°) = 360° - 185° = 175°$

b) $|\sphericalangle CSB| = ?$

$|\sphericalangle CSA| = 125°$

$|\sphericalangle ASB| = 80°$

$|\sphericalangle CSB| = 125° + 80° = 205°$

c) $|\sphericalangle BSC| = ?$

$|\sphericalangle BSA| = 230°$

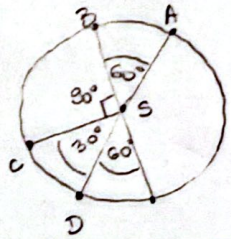
$|\sphericalangle CSA| = 90°$

$|\sphericalangle BSC| = 230° - 90° = 140°$

⑨ a) $|\sphericalangle CSD| = 60°$

b) $|\sphericalangle CSD| = 60°$ (vršni kutovi)

c) $|\sphericalangle CSD| = 30°$



⑩ a) $r = 5\,cm$

$o = ?$

$o = 2r\pi$

$o = 2 \cdot 5 \cdot 3.14$

$o = 31.4\,cm$

$P = r^2\pi$

$P = 5^2 \cdot 3.14$

$P = 5 \cdot 5 \cdot 3.14$

$P = 78.5\,cm^2$

b) $d = 0.6\,dm$

$o = ?$

$o = d\pi$

$o = 0.6 \cdot 3.14$

$o = 1.884\,dm$

$r = \frac{d}{2} = \frac{0.6}{2} = 0.3\,dm$

$P = r^2\pi$

$P = 0.3^2 \cdot 3.14$

$P = 0.2826\,dm^2$

c) $r = 2\frac{1}{5}\,m = 2.2\,m$

$o = ?$

$o = 2r\pi$

$o = 2 \cdot 2\frac{1}{5} \cdot 3.14$

$o = 2 \cdot \frac{11}{5} \cdot 3.14$

$o = \frac{69.08}{5}$

$o = 13.816\,m$

$P = r^2\pi$

$P = 2.2^2 \cdot 3.14$

$P = 15.1976\,m^2$

11. a) $|SB| = 1.5\,cm$ (polumjer)

b) $|AC| = 2 \cdot 1.5\,cm = 3\,cm$ (promjer)

c) $|\sphericalangle ASB| = 60°$ ($\triangle ABS$ je jednakostranični, $\overline{SB}$ i $\overline{SA}$ su iste veličine, $\sphericalangle SAB = 60°$);

d) $|\sphericalangle ASC| = 180°$ (ispruženi kut, leži na promjeru)

e) $|\sphericalangle BSC| = 120°$ ($|\sphericalangle ASC| - |\sphericalangle ASB| = 180° - 60° = 120°$)

*) $|\sphericalangle ABC| = 90°$ ($|\sphericalangle SBC| = |\sphericalangle SCB| = 30°$, $|\sphericalangle ABS| = 60°$)

12. $o = 43.96\,dm$

$d = ?$

$o = d\pi$

$d = \frac{o}{\pi}$

$d = \frac{43.96}{3.14}$

$d = 14\,dm$

13. $o = 18.84\,dm$

$r = ?$

$o = 2r\pi$

$r = \frac{o}{2\pi}$

$r = \frac{18.84}{2 \cdot 3.14}$

$r = \frac{18.84}{6.28}$

$r = 3\,cm$

14. $N = 200$

$o_{200} = 439.6\,m$

$r = ?$

$o_N = N \cdot o_1$ ($o_1 \rightarrow$ opseg 1 okreta)

$o_N = N \cdot 2r\pi$

$439.6 = 200 \cdot 2 \cdot r \cdot 3.14$

$439.6 = 1256 \cdot r \;/ : 1256$

$\frac{439.6}{1256} = r$

$r = 0.35\,m$

15. $r_{Eva} = 4\,m$

$r_{karlo} = 3.2\,m$

a) $o_{1\,Eva} = ?$ $o_{1\,karlo} = ?$

$o_{1\,Eva} = 2r_{Eva} \cdot \pi = 2 \cdot 4 \cdot 3.14 = 25.12\,m$

$o_{1\,karlo} = 2r_{karlo} \cdot \pi = 2 \cdot 3.2 \cdot 3.14 = 20.096\,m$

b) $N = 88$

$o_{Eva} = 88 \cdot 25.12 = 2210.56\,m$

$o_{karlo} = 88 \cdot 20.096 = 1768.448\,m$

$o_{Eva} - o_{karlo} = 442.112\,m$

16. a) $r = 4\,cm$

$P_o = ?$

$P = r^2\pi$

$P = 4^2 \cdot 3.14$

$P = 50.24\,cm^2$

b) $d = 0.7\,dm$

$P_o = ?$

$d = 2r \Rightarrow r = \frac{d}{2} = 0.35\,dm$

$P = r^2 \cdot \pi$

$P = 0.35^2 \cdot 3.14$

$P = 0.38465\,dm^2$

c) $r = \frac{3}{4}\,m = 0.75\,m$

$P_o = ?$

$P_o = r^2\pi$

$P_o = 0.75^2 \cdot 3.14$

$P_o = 1.76625\,m^2$

(17) $P = 153.86\ m^2$

$r = ?$

$P = r^2 \cdot \pi$

$r^2 = \frac{P}{\pi}$

$r^2 = \frac{153.86}{3.14}$

$r^2 = 49 = r \cdot r$

$r = 7\ m$

(18) a) $o = 25.12\ cm$

$P = ?$

$o = 2r\pi$

$r = \frac{o}{2\pi}$

$r = \frac{25.12}{2 \cdot 3.14}$

$r = 4\ cm$

$P = r^2\pi$

$P = 4^2 \cdot 3.14$

$P = 50.24\ cm^2$

b) $o = 37.68\ cm$

$P = ?$

$o = 2r\pi$

$r = \frac{o}{2\pi}$

$r = \frac{37.68}{2 \cdot 3.14}$

$r = 6\ cm$

$P = r^2\pi$

$P = 6^2 \cdot 3.14$

$P = 113.04\ cm^2$

(19) a) $P = 113.04\ cm^2$

$o = ?$

$P = r^2\pi$

$r^2 = \frac{P}{\pi}$

$r^2 = \frac{113.04}{3.14}$

$r^2 = 36$

$r = 6\ cm$

$o = 2r\pi$

$o = 2 \cdot 6 \cdot 3.14$

$o = 37.68\ cm$

b) $P = 314\ dm^2$

$o = ?$

$P = r^2\pi$

$r^2 = \frac{P}{\pi}$

$r^2 = \frac{314}{3.14}$

$r^2 = 100$

$r = 10\ dm$

$o = 2r\pi$

$o = 2 \cdot 10 \cdot 3.14$

$o = 62.8\ dm$

(20) $r_1 = 4\ cm$

$r_2 = 6.2\ cm$

$o_{kv} = ?\ P_{kv} = ?$

$o_{kv} = o_1 + o_2$

$o_{kv} = 2r_1\pi + 2r_2\pi$

$o_{kv} = (2 \cdot 4 + 2 \cdot 6.2) \cdot 3.14$

$o_{kv} = 20.4 \cdot 3.14$

$o_{kv} = 64.056\ cm$

$P_{kv} = P_2 - P_1$ (jer je $r_2 > r_1$)

$P_{kv} = r_2^2\pi - r_1^2\pi = (r_2^2 - r_1^2) \cdot \pi$

$P_{kv} = (6.2^2 - 4^2) \cdot 3.14$

$P_{kv} = 22.44 \cdot 3.14$

$P_{kv} = 70.4616\ cm^2$

(22) $l = 2.512\ dm = 25.12\ cm$

$r = 9\ cm$

$\alpha = ?$

$l = \frac{r\pi\alpha}{180^\circ} \quad / \cdot 180^\circ$

$l \cdot 180^\circ = r\pi \cdot \alpha \quad / : (r\pi)$

$\frac{l \cdot 180^\circ}{r \cdot \pi} = \alpha$

$\alpha = \frac{\overset{8}{25.12} \cdot \overset{20}{180^\circ}}{\underset{1}{9} \cdot 3.14_{1}}$

$\alpha = 8 \cdot 20$

$\alpha = 160^\circ$

(21) a) $r = 24\,cm$

$l = ?$

$$l = \frac{r\pi \cdot \alpha}{180^\circ}$$

$$l = \frac{24 \cdot 3.14 \cdot 30^\circ}{180^\circ}$$

$$l = 12.56\,cm$$

b) $r = 6\,mm$

$\alpha = 40^\circ 15'$

$l = ?$

$$\alpha = 40^\circ 15' = \left(40 + \frac{15}{60}\right)^\circ = (40 + 0.25)^\circ = 40.25^\circ$$

$$l = \frac{r\pi\alpha}{180^\circ}$$

$$l = \frac{6 \cdot 3.14 \cdot 40.25^\circ}{180^\circ}$$

$$l = 4{,}21\,mm$$

(23) $r = 9\,mm$

$\alpha = 150^\circ$

$P_{K1} = ?$

$$P_{K1} = \frac{r^2\pi \cdot \alpha}{360^\circ}$$

$$P_{K1} = \frac{9^2 \cdot 3.14 \cdot 150^\circ}{360^\circ}$$

$$P_{K1} = \frac{81 \cdot 3.14 \cdot 5}{12}$$

$$P_{K1} = 105.975\,mm^2$$

(24) $r = 6\,cm$

$P_{K1} = 75.36\,cm^2$

$\alpha = ?$

$$P_{K1} = \frac{r^2\pi \cdot \alpha}{360^\circ} \quad / \cdot 360^\circ$$

$$P_{K1} \cdot 360^\circ = r^2\pi \cdot \alpha \quad / : (r^2\pi)$$

$$\frac{P_{K1} \cdot 360^\circ}{r^2\pi} = \alpha$$

$$\alpha = \frac{75.36 \cdot 360^\circ}{6^2 \cdot 3.14}$$

$$\alpha = \frac{75.36 \cdot 360^\circ}{36 \cdot 3.14}$$

$$\alpha = 24 \cdot 10$$

$$\alpha = 240^\circ$$

(25) $r = 9\,cm$

$l = 14.13\,cm$

$P_{K1} = ?$

$$l = \frac{r\pi \cdot \alpha}{180^\circ}$$

$$\alpha = \frac{l \cdot 180^\circ}{r\pi}$$

$$\alpha = \frac{14.13 \cdot 180^\circ}{9 \cdot 3.14}$$

$$\alpha = 4.5 \cdot 20^\circ$$

$$\alpha = 90^\circ$$

$$P_{K1} = \frac{r^2\pi \cdot \alpha}{360^\circ}$$

$$P_{K1} = \frac{9^2 \cdot 3.14 \cdot 90^\circ}{360^\circ}$$

$$P_{K1} = \frac{81 \cdot 3.14}{4}$$

$$P_{K1} = 63.585\,cm^2$$

(26) $P_{K1} = 84{,}78\,cm^2$

$\alpha = 120^\circ$

$r = ?$

$$P_{K1} = \frac{r^2\pi \cdot \alpha}{360^\circ}$$

$$84.78 = \frac{r^2 \cdot 3{,}14 \cdot 120^\circ}{360^\circ}$$

$$84{,}78 = \frac{r^2 \cdot 3.14}{3} \quad / \cdot 3$$

$$254.34 = r^2 \cdot 3{,}14 \quad / : 3.14$$

$$81 = r^2$$

$$r \cdot r = 81$$

$$r = 9\,cm$$

(27.) $l = 33.912\ dm$

$\alpha = 72°$

$r = ?$

$l = \frac{r \pi \alpha}{180°}$

$r = \frac{l \cdot 180°}{\pi \cdot \alpha}$

$r = \frac{33.912 \cdot 180°}{3.14 \cdot 72°}$

$r = \frac{6104.16}{226.08}$

$r = 27\ dm$

(28.) a) $r_1 = 3cm$

$r_2 = 4cm$

$c = 15cm$

$O = ?\ P = ?$

Potrebno je prepoznati 2 polukruga i 1 pravokutan trokut!

$O = \frac{O_1}{2} + \frac{O_2}{2} + c$ — hipotenuza pravokutnog trokuta

polukružnice (ili duljine kružnih lukova nad kutovima 180°)

$O = \frac{2 r_1 \pi}{2_1} + \frac{2 r_2 \pi}{2_1} + c$

$O = 3 \cdot 3.14 + 4 \cdot 3.14 + 15$

$O = 36.98\ cm$

$P = \frac{P_1}{2} + \frac{P_2}{2} + P_\Delta$ → površina pravokutnog trokuta

površine polukrugova

$P = \frac{r_1^2 \pi}{2} + \frac{r_2^2 \pi}{2} + \frac{a \cdot b}{2}$

$P = (3^2 \cdot 3.14 + 4^2 \cdot 3.14 + 6 \cdot 8) \cdot \frac{1}{2}$

$P = 126.5 \cdot \frac{1}{2}$

$P = 63.25\ cm^2$

(28.) b) $r = 2cm$

$a = 3cm$

$O = ?,\ P = ?$

potrebno je prepoznati dvije četvrtine kruga (koje zajedno čine polukrug); pravokutnik!

$O = \frac{o}{2} + 2 \cdot a$

$O = \frac{2 r \pi}{2} + 2 \cdot a$

$O = 2 \cdot 3.14 + 2 \cdot 3$

$O = 12.28\ cm$

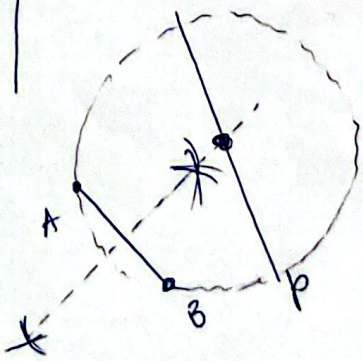
$P = \frac{P_o}{2} + a \cdot b$

$P = \frac{r^2 \pi}{2} + a \cdot b$

$P = \frac{2^2 \cdot 3.14}{2} + 2 \cdot 3$

$P = 12.28\ cm^2$

(6.) (skica)



(29.) i (30.) → riješeni na satu!